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EXACT ELASTODYNAMIC ANALYSIS OF SOME FRACTURE SPECIMEN MODELS INVOLVING STRIP GEOMETRIES

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Abstract—A procedure for analysing a class of transient elastodynamic crack problems is presented. These problems model certain experimental situations which can be used to infer fracture toughness values for materials under stress-wave loadings. In particular, the present analysis provides exact expressions for the elastodynamic stress intensity factor at the tip of a long external crack in a strip-like body whose lateral (upper and lower) boundaries are parallel to the crack line. Plane stress/strain conditions are assumed to prevail. In this class of problems, the crack may be situated asymmetrically with respect to the mid-strip line, and various dynamic loadings are considered, including crack face tractions and lateral face displacements. The loadings considered will have an arbitrary time dependence, but will be spatially uniform. The problem analysis is based on integral transforms and an asymptotic usage of the Wiener–Hopf technique. Two useful cases are considered in detail as examples; the symmetrically cracked strip with traction-free lateral boundaries under sudden crack face pressure, and the symmetrically cracked strip with suddenly displaced shear-free lateral boundaries.

1. INTRODUCTION

The present study lies within the framework of elastodynamic fracture mechanics and, in particular, is concerned with a class of model problems for stress-wave *diffraction* by stationary cracks. In this class, a typical plane stress/strain situation involves a planar crack in a nominally elastic body under the action of dynamically applied loads on the boundary or the crack faces. When a wave disturbance reaches the crack edge, a non-uniform scattered field radiates out behind longitudinal and shear wavefronts. Here, the constitutive equations and the equations of motion lead to *hyperbolic* governing equations, so that *transient* wave fields are anticipated. Such fields, of course, differ markedly from corresponding equilibrium (static) fields. An elastodynamic analysis usually aims at determining the crack tip stress intensity factor as a function of time and loading/geometry/material parameters. This information may, in turn, be utilized to quantify the material *resistance* to the onset of brittle fracture in dynamically loaded structures.

Some basic theoretical and analytical discussions pertinent to the above situations have been summarized by Achenbach (1971), Achenbach and Brock (1975), Brock (1975), Chen and Sih (1977), Atkinson (1977, 1986) and Freund (1990). Even more recent developments include work by Kundu and Mal (1981), Brock (1982, 1985, 1992, 1993), Keer *et al.* (1984), Kundu (1986), Freund (1987), Jiang and Knowles (1989), Georgiadis (1993), Georgiadis *et al.* (1991), Librescu and Shalev (1992) and Georgiadis and Brock (1993), among others. Experimental studies concerning cracks in a stress-wave environment also exist, e.g. Ravi-Chandar and Knauss (1982, 1984), Sukere and Sharpe (1983), Homma *et al.* (1983), Theocaris and Georgiadis (1983, 1984), Zehnder and Rosakis (1990) and Rossmanith and Knasmillner (1991).

The aforementioned theoretical studies are, indeed, greatly motivated by experimental evidence suggesting that the stress intensity factor (SIF) may provide a *one-parameter* representation of the load level near the edge of a dynamically loaded crack. The usual Irwin criterion [see e.g. Irwin (1960), Barenblatt (1962), Freund (1990)] may then be applied to the study of fracture initiation under stress-wave loading. That is, crack growth will commence if the dynamic SIF reaches a certain value (dynamic fracture toughness). This postulate does not, of course, imply that the dynamic fracture toughness is independent of loading rate, or that dynamic effects do not influence the fracture resistance in other ways. However, these issues must be resolved mainly through experiment.

Therefore, the elastodynamic analysis of certain crack configurations used for fracture toughness determination is of definite practical importance. Specifically, an analysis of *fracture specimens* can provide a theoretical expression for the SIF, which, in light of pertinent experimental measurements, can yield the material dynamic fracture toughness.

A fracture specimen which is particularly convenient for dynamic studies of the aforementioned type is a cracked strip (Fig. 1), i.e. a slab of material of width $(b+d)$ containing a long crack extending parallel to the strip lateral faces. In the general case, the crack line is not equidistant from the strip faces. A plane stress/strain mathematical idealization of this problem treats a domain $-\infty < x < \infty$, $-d < y < b$, with a crack existing along $-\infty < x < 0$, $y = 0$. This is the configuration that is to be considered here. The crack is loaded *dynamically* either by tractions applied directly to its faces or by an incident pulse created on the lateral boundary (boundaries) through some impact process. By superposition (due to the assumed linearity of governing equations) these two loading processes are equivalent so we shall deal only with crack face tractions, as they are especially amenable to the chosen solution scheme. The tractions may have an arbitrary time variation but are uniform in the spatial variable x. The solution scheme is based on *integral transform* analysis and an asymptotic use of the *Wiener–Hopf* (W–H) technique. Specifically, the transformed singular term of the crack tip stress field is found to depend only on expressions for the split W–H kernel valid for very small and very large values of the two-sided Laplace transform variable. Thus, an *asymptotic* kernel splitting is easily obtained, and inversions of the one- and two-sided Laplace transform follow without much difficulty.

The procedure is demonstrated by considering in detail the particular case of a symmetrical strip with traction-free lateral boundaries and crack face impact loading. In a subsequent section, a situation involving displacement controlled conditions will be presented, namely the symmetrical strip with shear-free suddenly displaced lateral boundaries. Finally, some generalizations of the procedure will briefly be discussed.

In concluding this Introduction some closely related works involving elastodynamics of cracked strips should be noted. Nilsson (1972) studied the steady-state propagation of a semi-infinite crack in a strip-like body. Nilsson (1975) also gave the solution of the anti-plane shear analogue of the present plane stress/strain problem, whereas Nilsson (1973) studied a transient plane stress/strain configuration by a path independent integral approach. The latter work is of particular interest here since it involved an alternate but less general solution approach. Similar steady-state or transient problems were also treated by Atkinson (1975, 1977), Popelar and Atkinson (1980), Georgiadis and Theocaris (1985), Georgiadis (1986), Freund (1990) and Marder (1991). It should be mentioned that these solutions for cracked strips also involve spatially *uniform* loading. A solution to such a

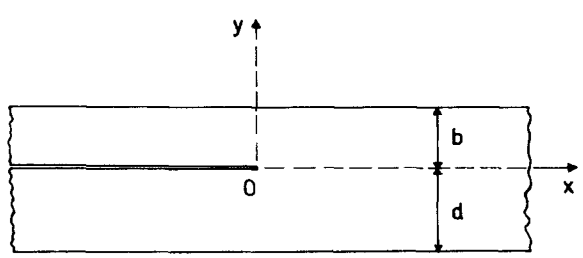

Fig. 1. A cracked strip of elastic material under uniform stress-wave loading.

problem which deals with *non-uniform* loading has, in fact, been obtained by Georgiadis and Brock (1993) by means of a novel function–theoretic technique.

2. GOVERNING EQUATIONS AND INTEGRAL TRANSFORMS

The material response assumed in the present analysis is linear elastodynamic. Also, two-dimensional bodies under in-plane loadings are considered and thus a plane stress/strain formulation is relevant. Therefore, with respect to an (x, y) Cartesian coordinate system, the governing equations for such states are written as

$$u_x = \frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial y} \tag{1a}$$

$$u_y = \frac{\partial \varphi}{\partial y} - \frac{\partial \psi}{\partial x} \tag{1b}$$

$$\sigma_x = \lambda \nabla^2 \varphi + 2\mu \left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \psi}{\partial x\, \partial y} \right) \tag{2a}$$

$$\sigma_y = \lambda \nabla^2 \varphi + 2\mu \left(\frac{\partial^2 \varphi}{\partial y^2} - \frac{\partial^2 \psi}{\partial x\, \partial y} \right) \tag{2b}$$

$$\tau_{xy} = \mu \left(2 \frac{\partial^2 \varphi}{\partial x\, \partial y} - \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) \tag{2c}$$

$$\nabla^2 \varphi = a_{\mathrm{L}}^2 \frac{\partial^2 \varphi}{\partial t^2}, \qquad \nabla^2 \psi = a_{\mathrm{T}}^2 \frac{\partial^2 \psi}{\partial t^2}, \tag{3a, b}$$

where (u_x, u_y) and $(\sigma_x, \sigma_y, \tau_{xy})$ are the components of the displacement vector and stress tensor, φ and ψ are the Lamé potentials, λ and μ are the Lamé constants of the material, $a_{\mathrm{L}} \equiv 1/c_{\mathrm{L}}$ and $a_{\mathrm{T}} \equiv 1/c_{\mathrm{T}}$ are the longitudinal (dilatational) and transverse (shear) wave slownesses, c_{L} and c_{T} are elastic wave velocities defined in terms of the material constants and the mass density, ρ, and $\nabla^2 = (\partial^2/\partial x^2) + (\partial^2/\partial y^2)$. All field quantities above are functions of the spatial variables x, y and the time variable t.

For convenience in the subsequent analysis, the x and t dependence in the governing equations and the boundary/initial conditions are suppressed through use of one- and two-sided Laplace transforms. The transforms and their corresponding inversion operations are

$$\bar{F}(x, y, s) = \int_0^{\infty} F(x, y, t)\, \mathrm{e}^{-st}\, \mathrm{d}t \tag{4a}$$

$$F(x, y, t) = \frac{1}{2\pi i} \int_{\mathrm{Br}} \bar{F}(x, y, s)\, \mathrm{e}^{st}\, \mathrm{d}s \tag{4b}$$

and

$$\bar{F}^*(p, y, s) = \int_{-\infty}^{\infty} \bar{F}(x, y, s)\, \mathrm{e}^{-spx}\, \mathrm{d}x \tag{5a}$$

$$\bar{F}(x, y, s) = \frac{s}{2\pi i} \int_{\mathrm{Br}} \bar{F}^*(p, y, s)\, \mathrm{e}^{spx}\, \mathrm{d}p, \tag{5b}$$

where Br denotes the Bromwich path in pertinent complex planes.

Application of the transforms (4a) and (5a) successively to eqns (1)–(3), gives the double transformed field quantities and equations

$$\bar{u}_x^* = (sp)\bar{\varphi}^* + \frac{\mathrm{d}\bar{\psi}^*}{\mathrm{d}y} \tag{6a}$$

$$\bar{u}_y^* = \frac{\mathrm{d}\bar{\varphi}^*}{\mathrm{d}y} - (sp)\bar{\psi}^* \tag{6b}$$

$$\bar{\sigma}_x^* = (\lambda+2\mu)(sp)^2\bar{\varphi}^* + \lambda\frac{\mathrm{d}^2\bar{\varphi}^*}{\mathrm{d}y^2} + 2\mu(sp)\frac{\mathrm{d}\bar{\psi}^*}{\mathrm{d}y} \tag{7a}$$

$$\bar{\sigma}_y^* = \lambda(sp)^2\bar{\varphi}^* + (\lambda+2\mu)\frac{\mathrm{d}^2\bar{\varphi}^*}{\mathrm{d}y^2} - 2\mu(sp)\frac{\mathrm{d}\bar{\psi}^*}{\mathrm{d}y} \tag{7b}$$

$$\bar{\tau}_{xy}^* = \mu\left[2(sp)\frac{\mathrm{d}\bar{\varphi}^*}{\mathrm{d}y} - (sp)^2\bar{\psi}^* + \frac{\mathrm{d}^2\bar{\psi}^*}{\mathrm{d}y^2}\right] \tag{7c}$$

$$\frac{\mathrm{d}^2\bar{\varphi}^*}{\mathrm{d}y^2} - \gamma_L^2\bar{\varphi}^* = 0, \qquad \frac{\mathrm{d}^2\bar{\psi}^*}{\mathrm{d}y^2} - \gamma_T^2\bar{\psi}^* = 0, \tag{8a, b}$$

where all transformed quantities are functions of (p, y, s), and

$$\gamma_L = s(a_L^2 - p^2)^{1/2}, \qquad \gamma_T = s(a_T^2 - p^2)^{1/2}. \tag{9a, b}$$

The behavior of the functions γ_j (j = L, T) in the cut p-plane is shown in Fig. 2.

The ordinary differential eqns (8a, b) give the solutions

$$\bar{\varphi}^* = \Phi_1(s, p)\cdot e^{\gamma_L y} + \Phi_2(s, p)\cdot e^{-\gamma_L y} \tag{10a}$$

$$\bar{\psi}^* = \Psi_1(s, p)\cdot e^{\gamma_T y} + \Psi_2(s, p)\cdot e^{-\gamma_T y}, \tag{10b}$$

where Φ_1, Φ_2, Ψ_1 and Ψ_2 are arbitrary functions. These solutions, along with eqns (6) and (7), will be utilized in subsequent analysis.

3. STRIP UNDER CRACK FACE IMPACT LOADING

3.1. *Problem statement*

Consider an elastic body in the form of an infinitely long strip occupying the region $(-\infty < x < \infty, -b < y < b)$ and containing a stationary semi-infinite crack situated along the plane $(-\infty < x < 0, y = 0)$. The material is stress-free and at rest everywhere for $t < 0$.

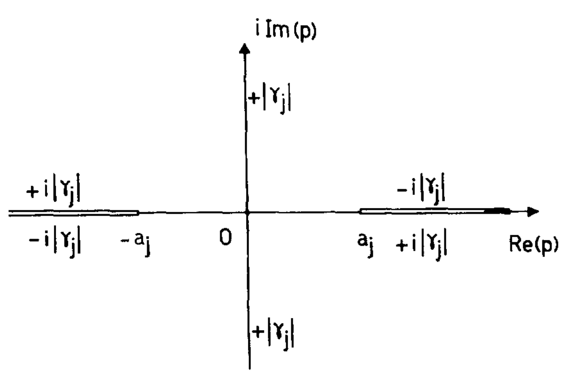

Fig. 2. The cut complex p-plane for the functions γ_j (j = L, T).

At $t = 0$, the crack faces are subjected to a suddenly applied, spatially uniform pressure of magnitude σ_0. Thus, the crack face traction behaves as a step function of time. This type of loading can, in fact, be obtained experimentally by an electromagnetic device (Ravi-Chandar and Knauss, 1984), but there are also other means to obtain it (Georgiadis, 1985). Moreover, the SIF for more general time dependence of loading can be obtained from the present solution by *convolution*.

Because of symmetry with respect to the plane $y = 0$, the problem can be viewed as a half-strip problem with the material occupying the region $(-\infty < x < \infty,\ 0 < y < b)$. Then, the associated initial/boundary value problem must satisfy the following conditions.

3.1.1. *Boundary and initial conditions.*

$$\sigma_y(x,b,t) = 0 \quad \text{for} \quad -\infty < x < \infty, \tag{11a}$$

$$\tau_{xy}(x,b,t) = 0 \quad \text{for} \quad -\infty < x < \infty \tag{11b}$$

$$\sigma_y(x,0,t) = -\sigma_0 \mathrm{H}(t) \quad \text{for} \quad -\infty < x < 0 \tag{11c}$$

$$\tau_{xy}(x,0,t) = 0 \quad \text{for} \quad -\infty < x < \infty \tag{11d}$$

$$u_y(x,0,t) = 0 \quad \text{for} \quad 0 < x < \infty \tag{11e}$$

$$\varphi(x,y,0) = \partial\varphi(x,y,0)/\partial t = \psi(x,y,0) = \partial\psi(x,y,0)/\partial t = 0, \tag{11f}$$

where $\mathrm{H}(\cdot)$ is the Heaviside step function.

3.1.2. *Edge conditions.*

$$\sigma_y(x,0,t) = o(1/x) \quad \text{for} \quad x \to 0^+ \tag{12a}$$

$$u_y(x,0,t) = O(1) \quad \text{for} \quad x \to 0^-, \tag{12b}$$

which guarantee that the *near tip* stress and displacement fields will not be so singular as to correspond to sources of radiated energy. Furthermore, on the basis of fracture mechanics considerations or by exact asymptotic analysis [see e.g. Barenblatt (1962), Karp and Karal (1962), Barber (1992), Georgiadis and Barber (1993)], it can be shown that $\sigma_y(x,0,t) \sim x^{-1/2}$ for $x \to 0^+$, and $u_y(x,0,t) \sim x^{1/2}$ for $x \to 0^-$. However, eqns (12a, b) are still sufficient conditions for applying Liouville's theorem in subsequent steps of our analysis.

3.1.3. *Finiteness conditions at remote regions.*

$$|\bar{\sigma}_y(x,0,s)| < A \exp(-p_\Sigma x) \quad \text{for} \quad x \to +\infty \tag{13a}$$

$$|\bar{u}_y(x,0,s)| < B \quad \text{for} \quad x \to -\infty, \tag{13b}$$

where A, B and p_Σ are positive constants. These equations guarantee that the diffraction field at infinity consists of *outgoing* waves only. More specifically, eqn (13a) is a direct consequence of the asymptotic behavior of the Laplace transformed solution of the wave eqns (3), whereas eqn (13b) expresses the fact that the field is actually uniform for large negative x.

The objective here is the exact determination of the stress field near to the crack tip for the problem defined by eqns (1)–(3) and (11)–(13). We note, in passing, that the corresponding crack problem in the absence of boundaries, i.e. the impact loading of a semi-infinite crack in an *unbounded* solid, is a classical diffraction problem considered by de Hoop (1958) and also presented in well-known texts, e.g. Achenbach (1973) and Freund (1990).

3.2. *Analysis*

It is seen from the problem statement that the functions $\sigma_y(x,0,t)$ and $u_y(x,0,t)$ are unknown in the intervals $x>0$ and $x<0$, respectively. Let us define *half-line* transforms of their one-sided Laplace transforms

$$\Sigma^+(p,s) \equiv \int_0^\infty \bar{\sigma}_y(x,0,s)\,\mathrm{e}^{-spx}\,\mathrm{d}x \quad \text{for} \quad -\frac{p_\Sigma}{s} \leqslant \mathrm{Re}\,(p) \tag{14a}$$

$$\bar{\sigma}_y(x,0,s) = \frac{s}{2\pi i}\int_{\mathrm{Br}_1} \Sigma^+(p,s)\,\mathrm{e}^{spx}\,\mathrm{d}p \quad \text{for} \quad 0<x<\infty \tag{14b}$$

$$U^-(p,s) \equiv \int_{-\infty}^0 \bar{u}_y(x,0,s)\,\mathrm{e}^{-spx}\,\mathrm{d}x \quad \text{for} \quad \mathrm{Re}\,(p) \leqslant 0 \tag{15a}$$

$$\bar{u}_y(x,0,s) = \frac{s}{2\pi i}\int_{\mathrm{Br}_2} U^-(p,s)\,\mathrm{e}^{spx}\,\mathrm{d}p \quad \text{for} \quad -\infty<x<0, \tag{15b}$$

where, in light of conditions (13), $\Sigma^+(p,s)$ and $U^-(p,s)$ are analytic functions in the indicated half-planes. These as yet unknown functions are to be determined by the W–H method. The definitions (14) and (15), in conjunction with the transformed boundary conditions (11c, e), yield

$$\bar{\sigma}_y^*(p,0,s) = \Sigma^+(p,s) + \frac{\sigma_0}{s^2 p} \tag{16}$$

$$\bar{u}_y^*(p,0,s) = U^-(p,s). \tag{17}$$

Then applying eqns (4a) and (5a) to the boundary conditions (11) in light of eqns (6), (7), (10), (16) and (17), and eliminating the functions Φ_1, Φ_2, Ψ_1 and Ψ_2 from the resulting system of five equations produces the following W–H equation

$$\Sigma^+(p,s) + \frac{\sigma_0}{s^2 p} = \frac{\mu}{2(a_\mathrm{T}s)^2}K(p,s)\cdot U^-(p,s). \tag{18}$$

Here the kernel function K is given by

$$K = \frac{N}{D}, \tag{19}$$

where

$$\begin{aligned} N(p,s) = {} & 16(sp)^2\gamma_\mathrm{L}\gamma_\mathrm{T}(\gamma_\mathrm{T}^2-(sp)^2)^2 + 8(sp)^2\gamma_\mathrm{L}\gamma_\mathrm{T}\,\mathrm{e}^{-\gamma_\mathrm{T}b} \\ & \cdot[4(sp)^2\gamma_\mathrm{L}\gamma_\mathrm{T}\sinh(\gamma_\mathrm{L}b) - (\gamma_\mathrm{T}^2-(sp)^2)^2\cosh(\gamma_\mathrm{L}b)] \\ & -[4(sp)^2\gamma_\mathrm{L}\gamma_\mathrm{T} + (\gamma_\mathrm{T}^2-(sp)^2)^2]\,\mathrm{e}^{\gamma_\mathrm{L}b} \\ & \cdot[4(sp)^2\gamma_\mathrm{L}\gamma_\mathrm{T}\cosh(\gamma_\mathrm{T}b) + (\gamma_\mathrm{T}^2-(sp)^2)^2\sinh(\gamma_\mathrm{T}b)] \\ & +[4(sp)^2\gamma_\mathrm{L}\gamma_\mathrm{T} - (\gamma_\mathrm{T}^2-(sp)^2)^2]\,\mathrm{e}^{-\gamma_\mathrm{L}b} \\ & \cdot[4(sp)^2\gamma_\mathrm{L}\gamma_\mathrm{T}\cosh(\gamma_\mathrm{T}b) - (\gamma_\mathrm{T}^2-(sp)^2)^2\sinh(\gamma_\mathrm{T}b)] \end{aligned} \tag{20a}$$

$$\begin{aligned} D(p,s) = {} & \gamma_\mathrm{L}[4(sp)^2\gamma_\mathrm{L}\gamma_\mathrm{T}\sinh(\gamma_\mathrm{L}b)\cdot\cosh(\gamma_\mathrm{T}b) \\ & +(\gamma_\mathrm{T}^2-(sp)^2)^2\cosh(\gamma_\mathrm{L}b)\cdot\sinh(\gamma_\mathrm{T}b)]. \end{aligned} \tag{20b}$$

In view of eqns (14a) and (15a), it is noted that eqn (18) holds over a common region of analyticity defined by the strip $-(p_{\Sigma}/s) \leqslant \mathrm{Re}(p) \leqslant 0$.

The single eqn (18) can be made to yield equations for both $\Sigma^{+}(p,s)$ and $U^{-}(p,s)$ through a decoupling procedure. The first step in that procedure is a factorization of the kernel in the form

$$K(p,s) = K^{+}(p,s)\cdot K^{-}(p,s), \tag{21}$$

where $K^{+}(p,s)$ and $K^{-}(p,s)$ are *non-zero* and *analytic* in a right and left half-plane, respectively. Moreover, these two half-planes should overlap. As is well known [see e.g. Noble (1958)], such a factorization can be accomplished by using either the Cauchy theorem or infinite product forms. At present, however, neither approach has yielded results for the very complicated kernel considered here. Fortunately, as will be seen later, an asymptotic approach that requires only the kernel's forms for $p \to 0$ and $p \to \infty$ can be applied, so that exact *asymptotic* expressions for the cleavage stress $\sigma_y(x,0,t)$ for $x \to 0^{+}$ can be obtained.

Assuming now for the moment that the kernel factorization (21) has been obtained, a rearrangement in eqn (18) gives

$$\frac{\Sigma^{+}(p,s)}{K^{+}(p,s)} + \frac{\sigma_0}{s^2 p\cdot K^{+}(p,s)} = \frac{\mu}{2(a_{\mathrm{T}}s)^2}K^{-}(p,s)\cdot U^{-}(p,s). \tag{22}$$

The sum-splitting of the second term in the left hand side of eqn (22) required to complete the decoupling process can now be obtained by inspection as

$$G(p,s) \equiv \frac{\sigma_0}{s^2 p\cdot K^{+}(p,s)} = G^{+}(p,s)+G^{-}(p,s), \tag{23}$$

where

$$G^{+}(p,s) = \frac{\sigma_0}{s^2 p}\left[\frac{1}{K^{+}(p,s)} - \frac{1}{K^{+}(0,s)}\right] \tag{24a}$$

$$G^{-}(p,s) = \frac{\sigma_0}{s^2 p}\,\frac{1}{K^{+}(0,s)} \tag{24b}$$

and $G^{+}(p,s)$ is an analytic function in the same *right* half-plane with that where $K^{+}(p,s)$ is defined, while $G^{-}(p,s)$ is an analytic function in the *left* half-plane $\mathrm{Re}(p) < 0$. Equations (22) and (23), when combined, allow the final rearrangement of the W–H relation

$$\frac{\Sigma^{+}}{K^{+}} + G^{+} = \frac{\mu}{2(a_{\mathrm{T}}s)^2}K^{-}\cdot U^{-} - G^{-} \equiv J(p,s). \tag{25}$$

The above equation holds in a certain strip of analyticity of the complex p-plane. However, the first part of eqn (25) is defined and is analytic in a certain right half-plane, whereas the second part is defined and is analytic in a certain left half-plane. Because the half-planes overlap in the strip, $J(p,s)$ is, by analytic continuation, defined and analytic over the whole p-plane. In order to determine $J(p,s)$ it is necessary to establish *order relations* for large p and then apply the Liouville theorem. By use of the Abel/Tauber theorems (van der Pol and Bremmer, 1950) and use of the asymptotic behavior for $\sigma_y(x,0,t)$, $x \to 0^{+}$ and $u_y(x,0,t)$, $x \to 0^{-}$ noted earlier, we easily find the asymptotic results: $\Sigma^{+} \sim p^{-1/2}$ and $U^{-} \sim p^{-3/2}$ for large p. Moreover, it can also be shown that

$$\lim_{p\to 0} K(p,s) = -2(a_T^4/a_L)s^3 \tanh(a_L bs) \tag{26}$$

$$\lim_{p\to\infty} K(p,s) = i4(a_L^2 - a_T^2)s^3 p, \tag{27}$$

where p is taken along the pertinent Bromwich path. Then, on rewriting eqn (27) in the form

$$K(\infty, s) = 4(a_L^2 - a_T^2)s^3(\varepsilon^2 - p^2)^{1/2} \quad \text{with} \quad \varepsilon \to 0, \tag{28}$$

the asymptotic kernel factorization follows easily as

$$K^+(\infty, s) = [4(a_L^2 - a_T^2)s^3 p]^{1/2} \tag{29a}$$

$$K^-(\infty, s) = [4(a_L^2 - a_T^2)s^3(-p)]^{1/2}. \tag{29b}$$

Likewise, we also find

$$K^+(0,s) = K^-(0,s) = [-2(a_T^4/a_L)s^3 \tanh(a_L bs)]^{1/2}. \tag{30}$$

All this asymptotic behavior of the functions appearing in eqn (25), implies that $J(p,s) = o(1)$ as $p \to \infty$ and by Liouville's theorem, therefore, $J(p,s) = 0$. This result, in conjunction with eqn (25), leads to the relation

$$\lim_{p\to\infty} \Sigma^+(p,s) = \frac{\sigma_0}{s^2 p} \frac{K^+(\infty,s)}{K^+(0,s)}, \tag{31}$$

where we have omitted a term whose inversion gives non-singular stress at the crack tip in the physical plane.

Clearly, the key observation which led to this useful asymptotic result was that the full kernel form given by eqns (19) and (20) could be replaced by the easily factorized form (27).

3.3. *Asymptotic results*

In view of the Abel/Tauber theorem (van der Pol and Bremmer, 1950; Noble, 1958) which relates asymptotically functions and their transforms, the *singular* part of the cleavage stress, $\lim_{x\to 0^+} \sigma_y(x,0,t)$, can be calculated from the large p expression, $\lim_{p\to\infty} \Sigma^+(p,s)$. To this end, eqns (29a) and (30) are used to rewrite eqn (31) as

$$\lim_{p\to\infty} \Sigma^+(p,s) = \frac{\sigma_0(2a_L)^{1/2} a_T^{-2}(a_T^2 - a_L^2)^{1/2}}{s^{3/2}[\tanh(a_L bs)]^{1/2}} \frac{1}{(sp)^{1/2}}, \tag{32}$$

which can be inverted immediately through eqn (14b) to give

$$\lim_{x\to 0^+} \bar{\sigma}_y(x,0,s) = \frac{\sigma_0 \pi^{-1/2}(2a_L)^{1/2} a_T^{-2}(a_T^2 - a_L^2)^{1/2}}{s^{3/2}[\tanh(a_L bs)]^{1/2}} \frac{1}{x^{1/2}}. \tag{33}$$

In order to obtain $\sigma_y(x,0,t)$ as $x \to 0^+$, it remains to obtain the inverse one-sided Laplace transform of eqn (33). However, we observe *a priori* that for a small time [i.e. for large s, where $\tanh(a_L bs) \to 1$] the stress behaves like $t^{1/2}$ (which is the inverse of $s^{-3/2}$), a result similar to that for the elastodynamic problem of a semi-infinite crack under impact tractions in a body of infinite extent. After some time, however, reflections from the strip lateral boundaries will affect the latter stress behavior. These reflections are represented by the term $[\tanh(a_L bs)]^{-1/2}$ in eqn (33). Although the s dependence in eqn (33) appears simple, a relevant entry is not found in inversion tables. Instead, we adopt a standard procedure [e.g. Carrier and Pearson (1976)] and expand $[\tanh(a_L bs)]^{-1/2}$ in a converging series and then perform the one-sided Laplace transform inversion term by term. Thus, we first write

$$s^{-3/2}[\tanh(a_L bs)]^{-1/2} = s^{-3/2}(1+e^{-2a_L bs}+\tfrac{1}{2}e^{-4a_L bs}+\tfrac{1}{2}e^{-6a_L bs}+\tfrac{6}{16}e^{-8a_L bs}+\cdots) \tag{34}$$

and note from a table [e.g. Abramowitz and Stegun (1972)]

$$\mathscr{L}^{-1}(s^{-3/2}e^{-ks}) = (2/\pi^{1/2})(t-k)^{1/2}H(t-k), \tag{35}$$

where $\mathscr{L}^{-1}(\cdot)$ denotes the inversion operator, and k is a positive constant.

In light of these results, eqns (33), (34) and (35) give

$$\begin{aligned}\sigma_y(x,0,t) = {} & (2^{3/2}/\pi)\cdot\sigma_0 a_L^{1/2}a_T^{-2}(a_T^2-a_L^2)^{1/2}x^{-1/2} \\ & \cdot[t^{1/2}+(t-2a_L b)^{1/2}H(t-2a_L b)+\tfrac{1}{2}(t-4a_L b)^{1/2} \\ & \cdot H(t-4a_L b)+\tfrac{1}{2}(t-6a_L b)^{1/2}H(t-6a_L b) \\ & +\tfrac{6}{16}(t-8a_L b)^{1/2}H(t-8a_L b)+\cdots] \quad \text{as} \quad x\to 0^+,\end{aligned} \tag{36}$$

whereupon the stress intensity factor follows as

$$\begin{aligned}K_I(t) &\equiv \lim_{x\to 0^+}[(2\pi x)^{1/2}\sigma_y(x,0,t)] \\ &= (4/\pi^{1/2})\sigma_0 a_L^{1/2}a_T^{-2}(a_T^2-a_L^2)^{1/2}[\ldots].\end{aligned} \tag{37}$$

Here the expression in brackets is the same as that in eqn (36). Equation (37) is an *exact* expression for the SIF as a function of time and loading/geometry/material parameters. This formula is valid until the time of ten half-width traversals by the longitudinal wave generated by the crack face tractions, but, of course, one can continue the series expansion in eqn (34) and obtain the response for a longer time. Figure 3 illustrates the SIF behavior vs time, as given by eqn (37). The longitudinal stress-wave reflections at the strip lateral faces are clearly identified as discontinuous changes.

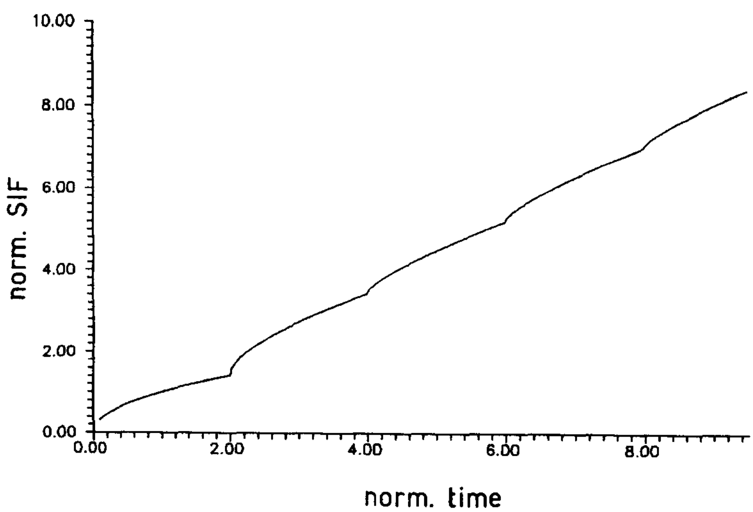

Fig. 3. Stress intensity factor history for a symmerically cracked strip under sudden crack face loading.

3.4. *Remarks*

One can immediately check that the expression in eqn (37) for $t < 2a_L b$, that is before any stress-wave reflection arrives from the lateral strip boundaries $y = \pm b$, is identical with the corresponding result obtained by de Hoop (1958) and Freund (1990) for the elastodynamic problem of suddenly applied crack face loading in an *unbounded* body. The latter analysis used the Cagniard–de Hoop technique in order to accomplish the one-sided Laplace transform inversion. In some contrast, the asymptotic approach utilized in the present analysis bypasses the Cagniard–de Hoop technique because it can, in effect, *decouple* the inversion of the two-sided Laplace transform from that of the one-sided transform, so long as *only* the stress field singular behavior is of concern.

It should be noted that only longitudinal wave reflections affect the SIF here; contributions by shear waves, which also propagate through the body, are absent. A simple explanation for this would be the symmetry of the problem, which imposes a purely opening mode (mode I) stress environment on the crack. Asymmetric situations will be discussed later on.

Another issue which merits comment is the long time limit of eqn (37). One sees that $K_I(t) \to \infty$ as $t \to \infty$, which means that no *static* limit exists for this problem. This conclusion is confirmed by elastostatic results for the double cantilever beam configuration (Fichter, 1983; Foote and Buchwald, 1985; Georgiadis and Papadopoulos, 1990). The de Hoop problem also, of course, has no static limit as noted by Baker (1962) and Freund (1990). However, if a loading pulse of *finite* duration is considered, instead of the H(t) dependence in eqn (11c), a different long time behavior of the solution should be anticipated.

The SIF for a more general crack face loading of the form $\sigma_y(x, 0^+, t) = -\sigma_0 f(t)$ for $-\infty < x < 0$, where $f(t)$ is assumed to be Laplace transformable, can be obtained for the symmetrical strip with traction-free lateral boundaries from the following expression

$$\bar{K}_I(s) = \frac{2\sigma_0 a_L^{1/2}(a_T^2 - a_L^2)^{1/2}}{a_T^2} \frac{\bar{f}(s)}{s^{1/2}[\tanh(a_L bs)]^{1/2}} \tag{38}$$

through Laplace transform inversion. The simple observation was made here that the term (σ_0/s) in the previous analysis can be replaced by $\sigma_0 \bar{f}(s)$, without affecting the basic solution procedure.

4. CRACKED STRIP WITH DYNAMICALLY DISPLACED LATERAL BOUNDARIES

4.1. *Problem statement*

Consider again an elastic body in the form of an infinitely long strip occupying the region $(-\infty < x < \infty, -b < y < b)$ and containing a stationary semi-infinite crack situated along the plane $(-\infty < x < 0, y = 0)$. The material is stress-free and at rest everywhere for $t < -(b/c_L)$. At $t = -(b/c_L)$, the lateral boundaries $-\infty < x < \infty$, $y = \pm b$ are given a time-varying, spatially uniform normal displacement $\pm u_0 f(t + b/c_L)$, but remain shear-free. The function $f(t)$ representing a general loading is assumed to be Laplace transformable. The crack faces remain traction-free, so that we are tacitly assuming that *no* crack closure occurs. This is true when the crack remains open always, which, in turn, should be the case when a positive, non-decreasing $u_0 f(t + b/c_L)$ is imposed. The interesting case $f(t) = H(t)$ will be discussed, in connection with the crack face displacement, after obtaining the general solution.

In order to facilitate the application of the W–H technique for this general problem, an auxiliary initial/boundary value problem is considered upon which a static crack problem should be superposed. These two problems are stated below, where the symmetry of the original problem with respect to the plane $y = 0$ is also exploited. The *transient* problem reads

$$u_y(x, b, t) = 0 \quad \text{for} \quad -\infty < x < \infty \tag{39a}$$

$$\tau_{xy}(x, b, t) = 0 \quad \text{for} \quad -\infty < x < \infty \tag{39b}$$

$$\sigma_y(x,0,t) = -\rho c_L u_0 f'(t) \quad \text{for} \quad -\infty < x < 0 \tag{39c}$$

$$\tau_{xy}(x,0,t) = 0 \quad \text{for} \quad -\infty < x < \infty \tag{39d}$$

$$u_y(x,0,t) = 0 \quad \text{for} \quad 0 < x < \infty \tag{39e}$$

$$\varphi(x,y,0) = \partial\varphi(x,y,0)/\partial t = \psi(x,y,0) = \partial\psi(x,y,0)/\partial t = 0, \tag{39f}$$

where $(\cdot)'$ denotes differentiation. The *static* problem has the following boundary conditions:

$$u_y(x,b) = u_0 f(t-b/c_L) \quad \text{for} \quad -\infty < x < \infty \tag{40a}$$

$$\tau_{xy}(x,b) = 0 \quad \text{for} \quad -\infty < x < \infty \tag{40b}$$

$$\sigma_y(x,0) = 0 \quad \text{for} \quad -\infty < x < 0, \tag{40c}$$

$$\tau_{xy}(x,0) = 0 \quad \text{for} \quad -\infty < x < \infty, \tag{40d}$$

$$u_y(x,0) = 0 \quad \text{for} \quad 0 < x < \infty. \tag{40e}$$

Clearly, the stress pulse $\rho c_L u_0 f'(t)$ appearing in eqn (39c) arises from the dynamic application of the uniform displacement along the lateral strip faces (Achenbach, 1973). The conditions (39a, b) are for a *smooth* boundary, and it is known (Achenbach, 1973) that a longitudinal wave will be reflected from it as a longitudinal wave of the same amplitude and sign. By superposition, the SIF for the original problem will be the sum of the SIFs for the problems (39) and (40). Moreover, the SIF formula for the problem (40) is a well-known result obtained by Rice (1967)

$$K_I^{\text{stat}} = \begin{cases} f(t-b/c_L)Eu_0/b^{1/2} & \text{for plane stress} \quad (41\text{a}) \\ f(t-b/c_L)Eu_0/(1-\nu^2)b^{1/2} & \text{for plane strain,} \quad (41\text{b}) \end{cases}$$

where E is the Young's modulus and ν is the Poisson's ratio.

Next, the solution to problem (39) will be briefly presented. Conditions (39) are also supplied with the edge and finiteness conditions stated in eqns (12) and (13).

4.2. *Brief analysis and results*

By following the lines of analysis presented earlier in this paper, we can produce the W–H equation

$$\Sigma^+(p,s) + \frac{\rho c_L u_0 \cdot \mathscr{L}[f'(t)]}{sp} = \frac{\mu}{(a_T s)^2} K(p,s) \cdot U^-(p,s), \tag{42}$$

where $\mathscr{L}[\cdot]$ denotes the direct one-sided Laplace transform operator, and the kernel is given by

$$K(p,s) = (1/\gamma_L)[4(sp)^2\gamma_L\gamma_T \coth(\gamma_T b) + (\gamma_T^2 - (sp)^2)^2 \coth(\gamma_L b)], \tag{43}$$

with the various material constants appearing in eqn (42) being defined immediately after eqn (3).

The asymptotic forms of the kernel in eqn (43) are obtained as

$$\lim_{p\to 0} K(p,s) = (a_T^4/a_L)s^3 \coth(a_L bs) \tag{44}$$

$$\lim_{p\to\infty} K(p,s) = i2(a_T^2 - a_L^2)s^3 p, \tag{45}$$

where p is taken along the pertinent Bromwich path. The above expressions enable the asymptotic kernel factorization as

$$K^{+}(\infty,s) = [2(a_{\mathrm{T}}^2-a_{\mathrm{L}}^2)s^3p]^{1/2} \tag{46}$$

$$K^{+}(0,s) = [(a_{\mathrm{T}}^4/a_{\mathrm{L}})s^3 \coth(a_{\mathrm{L}}bs)]^{1/2}. \tag{47}$$

Finally, the large p expression for the double transformed $\sigma_y(x,0,t)$ stress is found to be

$$\begin{aligned}\lim_{p\to\infty} \Sigma^{+}(p,s) &= \frac{\rho c_{\mathrm{L}} u_0 \mathscr{L}[f'(t)]}{sp}\frac{K^{+}(\infty,s)}{K^{+}(0,s)} \\ &= \frac{\rho c_{\mathrm{L}} u_0 \mathscr{L}[f'(t)]\cdot(2a_{\mathrm{L}})^{1/2}a_{\mathrm{T}}^{-2}(a_{\mathrm{T}}^2-a_{\mathrm{L}}^2)^{1/2}}{s^{1/2}[\coth(a_{\mathrm{L}}bs)]^{1/2}}\frac{1}{(sp)^{1/2}},\end{aligned} \tag{48}$$

which is readily inverted through eqn (14b) as

$$\lim_{x\to 0^{+}} \bar{\sigma}_y(x,0,s) = \frac{\rho c_{\mathrm{L}} u_0 \mathscr{L}[f'(t)](2a_{\mathrm{L}})^{1/2}a_{\mathrm{T}}^{-2}(a_{\mathrm{T}}^2-a_{\mathrm{L}}^2)^{1/2}}{(\pi s)^{1/2}[\coth(a_{\mathrm{L}}bs)]^{1/2}}\cdot\frac{1}{x^{1/2}}. \tag{49}$$

Also, the one-sided Laplace transformed stress intensity factor then follows from eqn (49) as

$$\begin{aligned}\bar{K}_{\mathrm{I}}(s) &\equiv \lim_{x\to 0^{+}} [(2\pi x)^{1/2}\bar{\sigma}_y(x,0,s)] \\ &= \frac{2\rho c_{\mathrm{L}} u_0 \mathscr{L}[f'(t)]\cdot a_{\mathrm{L}}^{1/2}a_{\mathrm{T}}^{-2}(a_{\mathrm{T}}^2-a_{\mathrm{L}}^2)^{1/2}}{s^{1/2}[\coth(a_{\mathrm{L}}bs)]^{1/2}}\end{aligned} \tag{50}$$

Next, upon expanding the term $[\coth(a_{\mathrm{L}}bs)]^{-1/2}$ in a converging series

$$[\coth(a_{\mathrm{L}}bs)]^{-1/2} = 1-\mathrm{e}^{-2a_{\mathrm{L}}bs}+\tfrac{1}{2}\mathrm{e}^{-4a_{\mathrm{L}}bs}-\tfrac{1}{2}\mathrm{e}^{-6a_{\mathrm{L}}bs}+\tfrac{6}{16}\mathrm{e}^{-8a_{\mathrm{L}}bs}-\cdots \tag{51}$$

the one-sided Laplace transform inversion of $\bar{K}_{\mathrm{I}}(s)$ is greatly facilitated, as explained earlier for the case of a strip under crack face impact loading and traction-free lateral boundaries.

As an example, the SIF for the auxiliary problem in the case of *impact* displacement loading, i.e. $f(t) = \mathrm{H}(t)$, can easily be obtained by using eqns (50), (51) and a Laplace transform table as

$$\begin{aligned}K_{\mathrm{I}}(t) &= (2/\pi^{1/2})\rho c_{\mathrm{L}} u_0 a_{\mathrm{L}}^{1/2} a_{\mathrm{T}}^{-2}(a_{\mathrm{T}}^2-a_{\mathrm{L}}^2)^{1/2} \\ &\cdot\left[\frac{1}{t^{1/2}}-\frac{\mathrm{H}(t-2a_{\mathrm{L}}b)}{(t-2a_{\mathrm{L}}b)^{1/2}}+\frac{1}{2}\frac{\mathrm{H}(t-4a_{\mathrm{L}}b)}{(t-4a_{\mathrm{L}}b)^{1/2}}\right. \\ &\left.-\frac{1}{2}\frac{\mathrm{H}(t-6a_{\mathrm{L}}b)}{(t-6a_{\mathrm{L}}b)^{1/2}}+\frac{6}{16}\frac{\mathrm{H}(t-8a_{\mathrm{L}}b)}{(t-8a_{\mathrm{L}}b)^{1/2}}-\cdots\right]\end{aligned} \tag{52}$$

whereupon the SIF for the original problem is

$$K_{\mathrm{I}}^{\mathrm{orig}} = K_{\mathrm{I}}^{\mathrm{stat}}+K_{\mathrm{I}}(t), \tag{53}$$

where $K_{\mathrm{I}}^{\mathrm{stat}}$ is provided by eqn (41) with $f(t-b/c_{\mathrm{L}}) = \mathrm{H}(t-b/c_{\mathrm{L}})$. Some observations on the partial solution (52) are then worthy of note. First, we see that $K_{\mathrm{I}}(t)\to\infty$ as $t\to 0$, which is a consequence of crack face loading by a pulse containing the Dirac delta function of time. We also note that $K_{\mathrm{I}}(t) \to 0$ as $t \to \infty$, so that, appropriately, $K_{\mathrm{I}}^{\mathrm{orig}}(t) \to K_{\mathrm{I}}^{\mathrm{stat}}$ as $t \to \infty$. It can also be shown that $K_{\mathrm{I}}(t)$ periodically takes on negative values, thus implying crack *closure*. We have, of course, excluded crack face contact in formulating the mathematical problem, but one should keep in mind that the mathematical crack considered here is merely an idealization of the *narrow slit* of finite thickness normally found in experimental situations involving pre-cracked specimens. Therefore, it is reasonable to assume that no contact actually takes place. Indeed, it has been found that usual stress-wave amplitudes of 0(10 MPa) produce crack face displacements of $0(10^{-6}$ m) [see e.g. the experimental results by Sukere and Sharpe (1983)]. Moreover, Brock (1982) did calculations for point-force loading of a semi-infinite crack in an unbounded solid and found crack surface displacements of $0(10^{-5}$ m). Brock *et al.* (1985) and DeGiorgi and Brock (1990) also found that the mathematical crack idealization worked well in representing dynamic crack response under impact even when crack closure occurred.

5. GENERALIZATIONS AND CONCLUDING REMARKS

The asymptotic approach introduced here to solve two problems of symmetrical cracked strips can be generalized to *nonsymmetrical* situations. As an illustration, the asymmetrically cracked strip shown in Fig. 1 is considered. The material is stress-free and at rest everywhere for $t < 0$. At $t = 0$, the crack faces are subjected to a dynamically applied, spatially uniform pressure of magnitude of $\sigma_0 f(t)$. The lateral strip faces remain always traction-free. The boundary conditions therefore take the form

$$\sigma_y(x, b, t) = \tau_{xy}(x, b, t) = 0 \quad \text{for} \quad -\infty < x < \infty \tag{54a}$$

$$\sigma_y(x, -d, t) = \tau_{xy}(x, -d, t) = 0 \quad \text{for} \quad -\infty < x < \infty \tag{54b}$$

$$\sigma_y(x, \pm 0, t) = -\sigma_0 f(t)\mathrm{H}(-x) + (1/2\pi i)\int_{\mathrm{Br}}\left[(s/2\pi i)\int_{\mathrm{Br}_1}\Sigma^+(p,s)\,\mathrm{e}^{spx}\,\mathrm{d}p\right]\mathrm{e}^{st}\,\mathrm{d}s$$
$$\text{for} \quad -\infty < x < \infty \tag{54c}$$

$$\tau_{xy}(x, \pm 0, t) = (1/2\pi i)\int_{\mathrm{Br}}\left[(s/2\pi i)\int_{\mathrm{Br}_1} T^+(p,s)\,\mathrm{e}^{spx}\,\mathrm{d}p\right]\mathrm{e}^{st}\,\mathrm{d}s \quad \text{for} \quad -\infty < x < \infty \tag{54d}$$

$$u_y(x, +0, t) - u_y(x, -0, t) = (1/2\pi i)\int_{\mathrm{Br}}\left[(s/2\pi i)\int_{\mathrm{Br}_2}[U^-_{\mathrm{up}}(p,s) - U^-_{\mathrm{lo}}(p,s)]\,\mathrm{e}^{spx}\,\mathrm{d}p\right]\mathrm{e}^{st}\,\mathrm{d}s$$
$$\text{for} \quad -\infty < x < \infty \tag{54e}$$

$$u_x(x, +0, t) - u_x(x, -0, t) = (1/2\pi i)\int_{\mathrm{Br}}\left[(s/2\pi i)\int_{\mathrm{Br}_2}[V^-_{\mathrm{up}}(p,s) - V^-_{\mathrm{lo}}(p,s)]\,\mathrm{e}^{spx}\,\mathrm{d}p\right]\mathrm{e}^{st}\,\mathrm{d}s$$
$$\text{for} \quad -\infty < x < \infty \tag{54f}$$

$$\varphi(x,y,0) = \partial\varphi(x,y,0)/\partial t = \psi(x,y,0) = \partial\psi(x,y,0)/\partial t = 0, \tag{54g}$$

where $\Sigma^+(p,s)$ is the double transform of the as yet unknown normal stress along $0 < x < \infty$, and $U^-_{\mathrm{up}}(p,s)$ and $U^-_{\mathrm{lo}}(p,s)$ are the double transformed normal displacements along $-\infty < x < 0$, also unknown, of the upper $(+0)$ and lower (-0) crack face, respectively. The function $\Sigma^+(p,s)$ has already been defined in eqn (14), whereas the functions $U^-_{\mathrm{up}}(p,s)$ and $U^-_{\mathrm{lo}}(p,s)$ can be identified with $U^-(p,s)$ in eqn (15). Similarly, $T^+(p,s)$ is the double transformed shear stress along $0 < x < \infty$, and $V^-_{\mathrm{up}}(p,s)$ and $V^-_{\mathrm{lo}}(p,s)$ are the double transformed tangential displacements along $-\infty < x < 0$ of the upper $(+0)$ and lower (-0) crack face, respectively. These are also unknown functions and are defined according to eqns (14) and (15). All the transformed functions in eqn (54) are analytic in pertinent half-planes of the complex p-plane. Finally, it is noted that conditions (54e, f) express *continuity* of the displacement along $0 < x < \infty$, whereas in writing eqn (54d) it is assumed that no shear loading is applied to the crack surfaces (however, such a loading can easily be introduced in the present formulation).

By now transforming eqn (54) with the operations (4a) and (5a), and inserting eqns (6), (7) and (10), a system of eight equations results containing as unknowns the functions Φ_1, Φ_2, Ψ_1, Ψ_2, T^+, $V_{\rm up}^-$, $V_{\rm lo}^-$, Σ^+, $U_{\rm up}^-$, and $U_{\rm lo}^-$. Elimination of the first seven functions produces a W–H equation of the form

$$\Sigma^+ + \frac{\sigma_0 \bar{f}(s)}{sp} = K(U_{\rm up}^- - U_{\rm lo}^-), \tag{55}$$

where the kernel $K(p,s)$ will contain $(\mu, \gamma_{\rm L}, \gamma_{\rm T}, b, d)$. Despite the anticipated complications in the form of K, one can, in principle, follow the asymptotic approach described previously and determine the near tip stress field. The shear stress $\tau_{xy}(x \to 0^+, 0, t)$ can be found in the same way by formulating a W–H equation containing T^+ and $(V_{\rm up}^- - V_{\rm lo}^-)$.

At this point we close this paper by observing, in summary, that an exact elastodynamic analysis for a class of fracture mechanics problems was performed. This analysis was based on integral transform theory, and relied on an asymptotic version of the Wiener–Hopf technique. The problems class considered was specifically chosen to model certain fracture mechanics specimens which are often utilized to obtain material *fracture toughness* under *stress-wave* loadings. It is hoped that the present analysis will prove to be useful in this connection.

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